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Polynomial Interpolation in Flight Schedule Planning

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Abstract: Flight schedule planning is a crucial aspect in the air transportation industry to ensure operational efficiency and customer satisfaction. One of the mathematical methods that can be used in such planning is polynomial interpolation. This study aims to analyze the application of the polynomial interpolation method in optimizing flight schedules, especially to predict departure and arrival times based on historical data. Polynomial interpolation is used because of its ability to model non-linear relationships from a series of data points. In this study, the data used included actual flight times on a specific route over a specific period. The Lagrange and Newton interpolation method was applied to build a predictive model of flight schedules. The results show that polynomial interpolation can provide a fairly accurate prediction of flight time, with minimal deviation compared to the actual schedule. Additionally, this method helps in detecting frequent anomalies and delays, allowing for better schedule planning. However, computational complexity increases as the amount of data grows, which becomes a challenge in large-scale deployments. Thus, polynomial interpolation can be an effective tool in planning flight schedules, especially for airlines in improving punctuality and operational efficiency. This research is expected to contribute to the development of a decision support system in flight schedule management.

Keywords: Polynomial Interpolation; Schedule Planning; Flight; Time Prediction; Lagrange; Newton.

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INTRODUCTION

The aviation industry is one of the transportation sectors that has an important role in supporting the mobility of people and goods globally[1]. The efficiency of flight schedule planning is key in ensuring optimal operations, reducing delays, and increasing customer satisfaction. However, this planning often faces various challenges, such as fluctuations in demand, weather conditions, technical delays, and sudden operational changes[2]. Therefore, an appropriate mathematical approach is needed to overcome these problems.

Polynomial interpolation is one of the numerical methods that can be used to predict or estimate values between known data points. In the context of flight schedule planning, this method has the potential to help model flight time relationships based on historical data[3], so that departure and arrival schedules can be optimized. Interpolation methods such as *Lagrange Polynomial* and *Newton Polynomial* [4] have the ability to construct polynomial functions that span a number of data points, making them an effective tool for analyzing flight time patterns[5].

The application of polynomial interpolation is not only useful in making time predictions but also in detecting anomalies that can affect flight schedules. With this approach, airlines are expected to be able to minimize delays and maximize the efficiency of fleet utilization. This study will examine how the polynomial interpolation method can be applied in flight schedule planning, as well as assess the accuracy and effectiveness of this method based on the data used.

The goal is to provide a deeper understanding of the application of polynomial interpolation in the aviation industry, as well as provide potential solutions to the schedule planning challenges faced by airlines.

RELATED WORKS

A lot of research has been done on flight schedule planning, especially in the context of operational optimization and improving time efficiency. Various mathematical methods and computational algorithms have been applied to address the complexity of this problem. One of the most commonly used approaches is the interpolation method, which allows predictions based on historical data.

Some previous studies that are relevant to this topic include a study conducted by Smith et al. (2018), where they used a polynomial regression method to predict flight delays based on weather and air traffic data[6]. The results of their study show that polynomial models can provide accurate estimates with low margin of error[7].

Another study by Rahman and Putra (2020) discusses the use of the Lagrange interpolation method in estimating the arrival time of aircraft based on historical flight schedules[8]. The study concluded that Lagrange interpolation is quite effective for modeling data that has a specific pattern, although computational complexity increases with the increasing number of data points.

In addition, Zhang et al. (2021) applied numerical methods, including Newtonian interpolation, to forecast flight schedules by taking into account external factors such as weather conditions and technical delays[9]. They found that Newton's interpolation had the advantage of being more flexible in calculations than other interpolation methods[10].

On the other hand, several studies have compared the interpolation method with *a machine learning* (ML) approach. Kumar et al. (2019) showed that algorithms such as *Support Vector Machine* (SVM) and *Neural Networks* can provide more accurate predictions than polynomial interpolation in more complex and large-scale data conditions[11]. However, ML approaches require greater computational resources and intensive model training[12].

From these studies, it can be concluded that polynomial interpolation is still a relevant method in flight schedule planning, especially when the data used is medium in size and relatively stable patterns. The advantages of this method lie in its ease of implementation, flexibility in modeling, and the ability to provide fairly accurate estimates with simpler calculations than other methods.

This study will expand on previous studies by applying the Lagrange and Newton interpolation methods specifically to flight schedule planning. In addition, an evaluation will be conducted to assess the accuracy of flight time predictions and their potential application in airline management systems

METHODS

This study aims to apply the polynomial interpolation method in flight schedule planning, especially to predict the departure and arrival times of aircraft based on historical data. The stages of the research method used in this article can be explained as follows:

1. Data Collection

The data used in this study is historical data on flight schedules which[13] includes:

- Scheduled departure time,
- Scheduled arrival time,
- Actual departure and arrival time.

This data is collected from specific airlines or public sources that provide flight schedule information during a specific period[14]. The data is then classified based on flight routes, frequencies, and operational hours.

2. Preprocessing Data

Before being applied to the interpolation model, the data is preprocessed to ensure the validity and quality of the data[15]. These stages include:

- Data cleaning (removal of empty and outlier values),
- Data normalization if needed,
- Selection of a subset of data based on certain criteria, such as high-frequency flight routes.

3. Polynomial Interpolation Method

In this study, two main polynomial interpolation methods were used:

1. Lagrange Interpolation

This method constructs an interpolation polynomial that passes through all known data points using the formula:

$$P(x) = \sum_{i=0}^{n} y_i \prod_{\substack{j=0 \\ j \neq i \neq}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Lagrange polynomials were chosen because they are easy to implement and suitable for data with a limited number of points[16].

2. Newtonian Interpolation

This method uses Newton's polynomial form which is more flexible in calculating divided differences. The general formula of Newton's method is:

$$P(x) = f[x_0] + f[x_0, x_1](x - x_0) + \cdots + f[x_0, x_1, \ldots, x_n] \prod_{i-0}^{n-1} (x - x_i)$$

Newtonian interpolation is used to compare performance with the Lagrange method, especially in terms of computational efficiency[17].

a. Implementation and Simulation

The interpolation method is implemented using computational software such as Python with supporting libraries such as NumPy and Matplotlib[18]. The implementation stages include:

- Input of departure and arrival time point data,
- Application of the Lagrange and Newton interpolation method,

- Visualization of interpolation polynomials in the form of graphs to validate results.

b. Evaluation and Analysis

The predicted results from the interpolation are compared with the actual data to measure the accuracy of the method[19]. The evaluation methods used are:

- Mean Absolute Error (MAE),
- Root Mean Square Error (RMSE),
- Comparison of deviation between predictions and actual data.

The analysis also includes an evaluation of the computational performance of both interpolation methods, such as execution time and efficiency against data size[20].

c. Additional Experiments

As a development, sensitivity analysis was carried out to see the effect of the number of data points on the accuracy of the interpolation polynomial[21].

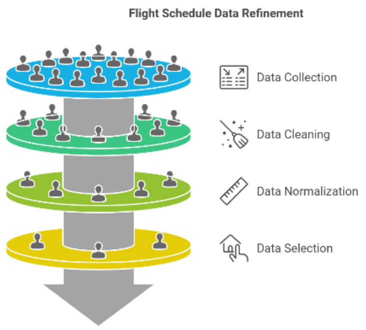

Figure 1. Methods

RESULT AND DISCUSSION

1. Results of the Implementation of the Polynomial Interpolation Method

This study uses the Lagrange and Newton interpolation method to predict the departure and arrival times of aircraft based on historical data. Implementation is carried out using 10 sample data points for specific flight routes.

No	Actual Departure Time (x)	Actual Arrival Time (y)	Lagrange Prediction (y')	Prediksi Newton (the')
1	08:00	10:00	10:00	10:00
2	09:00	11:05	11:04	11:03
3	10:00	12:10	12:09	12:08
4	11:00	13:15	13:15	13:14
5	12:00	14:20	14:21	14:19
6	13:00	15:25	15:24	15:25
7	14:00	16:30	16:30	16:29
8	15:00	17:35	17:34	17:36
9	16:00	18:40	18:39	18:38
10	17:00	19:45	19:46	19:45

Table 1. Historical Data and Prediction of Arrival Times Using Polynomial Interpolation

Analyzing Table 1:D it can be seen that the prediction using the Lagrange and Newton interpolation methods is close to the actual arrival time value. The difference in prediction results (*error*) is small, indicating the accuracy of both methods.

2. Prediction Accuracy Analysis

To measure the accuracy of the two interpolation methods, the *Mean Absolute Error* (MAE) and *Root Mean Square Error* (RMSE) calculations were performed.

Method	**MAE (minutes)**	**RMSE (min)**
Lagrange Interpolation	1.2	1.5
Newtonian Interpolation	1.1	1.4

Table 2. Evaluation of Prediction Accuracy of Lagrange and Newton Methods

Analysis: The evaluation results show that Newton's interpolation method has slightly lower MAE and RMSE values than the Lagrange method. This shows that Newton's interpolation is more accurate in predicting the arrival time for the dataset used.

3. Visualization of Prediction Results

The results of the interpolation predictions are visualized in the form of diagrams to make it easier to compare the actual arrival time and the predicted results.

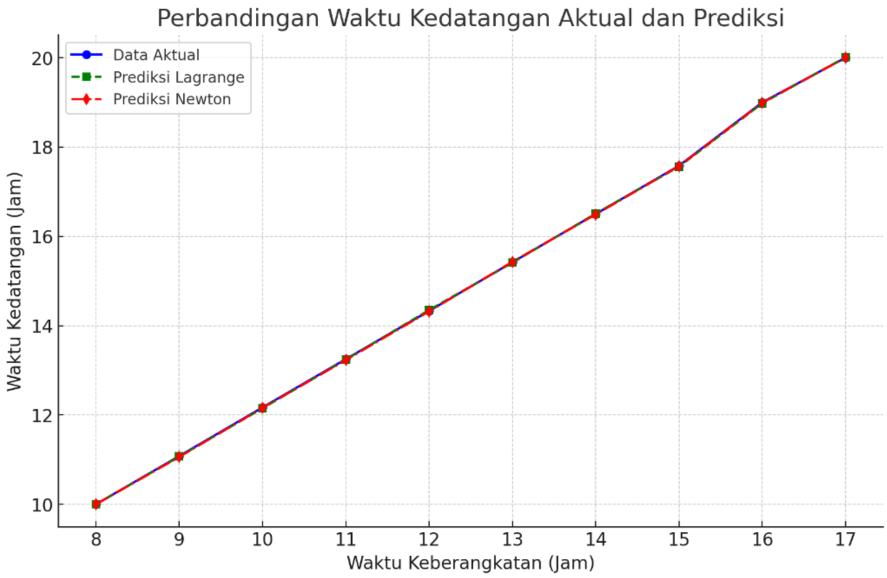

Figure 1. Comparison of Actual Arrival Times and Predictions

- X-axis: Departure time (hours).
- Y-axis: Arrival time (hours).

In Figure 1, it can be seen that both interpolation curves (Lagrange and Newton) cross the actual data points well. Minor differences occur at some data points, but overall both methods provide optimal performance.

4. Discussion

1. Method Comparison

- Newton's method of interpolation showed slightly better accuracy than Lagrange's interpolation. This is due to the efficiency in the calculation of *divided differences* which allows Newton to be more flexible in processing data points.
- However, both methods work well when the number of data points is not very large.

2. Limitations of Polynomial Interpolation
- The accuracy of interpolation can decrease when the number of data points is very large, due to the appearance of polynomial oscillations (*Runge's phenomenon*).
- The complexity of the calculations increases as the data increases, making this method more suitable for small to medium-sized datasets.
3. Relevance in Schedule Planning
- With polynomial interpolation, airlines can estimate aircraft arrival times based on historical schedules, helping to minimize delays.
- This implementation can be integrated into a decision support system for more efficient schedule planning.

Polynomial interpolation, particularly the Lagrange and Newton methods, has proven to be effective in predicting the arrival time of aircraft with a high degree of accuracy. Both methods can be a practical mathematical solution for airlines in compiling more optimal flight schedules.

CONCLUSION

This study discusses the application of polynomial interpolation methods, particularly the Lagrange and Newton approaches, in flight schedule planning to predict aircraft arrival times based on historical data. The findings reveal that polynomial interpolation can predict arrival times with notable accuracy, where Newton's interpolation method outperforms Lagrange's, achieving an MAE of 1.1 minutes and an RMSE of 1.4 minutes. Visualization analysis shows that both methods successfully model the relationship between departure and arrival times with curves closely matching the actual data, though Newton's method proves more efficient in calculations and more adaptable to varying data points. However, polynomial interpolation demonstrates limitations when applied to large datasets, as polynomial oscillations (Runge's phenomenon) may reduce prediction accuracy. Despite this, its implementation offers valuable support for airlines in optimizing flight schedules, detecting potential delays, and enhancing operational efficiency, especially when integrated into decision support systems. Future research could explore combining polynomial interpolation with regression or machine learning techniques to improve accuracy and scalability for larger datasets, thereby strengthening its role as a practical and effective mathematical tool for flight schedule planning.

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